Rich by accident: the second welfare theorem with a redundant asset under imperfect foresight

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Aim and Goal

this is a joint work with Shurojit Chatterji (Singapore Management U.)

- Consider a dynamic competitive market model with no uncertainty
- Assets are traded for saving/borrowing, and a redundant asset has no effect on consumption under perfect foresight.
  - a redundant asset = returns can be replicated by other assets
- One might assert robustness of PF, meaning that small deviations from PF have only small impacts on how goods are allocated.
- Robustness holds when there is no redundant asset, but when there is one, a minor deviation has a major impact on the allocation of goods.
- Thus we discover a powerful allocational implication of a redundant asset which has been overlooked in the rational expectations paradigm.

A general equilibrium model (Classic perfect competition)

the exchange economy: fundamentals

- There are 3 periods, $t = 0, 1, 2$
- A single non-storable good is available in every period.
- ♡ There is no uncertainty, and no friction
- ♣ H households, with additively separable utility function $\sum_t u_h(x^t)$
  - u_h is increasing and concave for every h
- household h is endowed with $e_h = (e_h^0, e_h^1, e_h^2) >> 0$

efficient allocations

- total supply is constant across time: $\sum_h e_h^t = 1$, $t = 0, 1, 2$
  - this is just for simplicity of exposition
- efficient allocation = perfect smoothing: every h consumes a constant share over time

Bond Markets

sequential markets - structure of markets in period t

- spot market for the good
- markets for bonds

assets - two kinds of discount bonds, zero net supply

1. S- bond, it matures in one period
    - the bond traded in period t pays out \$1 in period $t+1$
    - price q^t, bond holding of h : b_h^t
2. L- bond, matures in final period 2
    - it pays out \$1 in period 2,
    - can be traded in any peiod, price q_L^t, L-bond holding of h : l_h^t

- Note: they are identical in period 1, so $q^1 = q_L^1$

Temporary Equilibrium

Budget constaint

♡ Being a price taker, a household will forecast prices and variables to choose for a utility maximizing trading plan

- The forecast dynamic budget constraint in period 0:

$$
\begin{aligned}
p^0x^0 + q^0b^0 + q_L^0 l^0 &\leq p^0 e_h^0, \\
p^1x^1 + q^1b^1 + q_L^1 l^1 &\leq p^1 e_h^1 + b^0 + q_L^1 l^0, \\
p^2x^2 &\leq p^2 e_h^2 + b^1 + l^1,
\end{aligned}
$$

- similarly in period 1 (but forecasts may be freely updated)

Temporary Equilibrium (TE)

markets are in TE if demand = supply in all markets for every period t (there is no restrictions on forecasting method)

No Arbitrage Condition - redundancy of L-bond

NA Condition in period 0: $q_L^0 = q^0 q^1$

- if it fails, there is "free lunch" - a positive return at no cost
- if it holds, L- bond can be replicated by trading S bonds.
  - ♡ the dynamic budget constraint can be reduced to a single budget, utilizing the suitably discounted prices

Implication on forecasts of a household in TE

♡ $q_L^0 = q^0 q^1$ must hold, or else it finds free lunch - inconsistent with TE

♠ The forecast budget constraint can be reduced to a single budget

$$p^0 x^0 + p^1 x^1 + p^2 x^2 \leq p^0 e_h^0 + p^1 e_h^1 + p^2 e_h^2 \tag{1}$$

Two types of ETE

We classify efficient temporary equilirbia (ETE) by NA

Category 1: observed market prices satisfy NA: $q^0_L = q^0 q^1$

- special case is perfect foresight (PFE)
- any ETE is close to PFE if prices are close to PFE prices

◇ proof idea:

1. consumption x_h satisfies the budget constraint with respect to the observed prices
2. the single budget (1) converges to PFE budget, so x_h must be on PFE budget set in the limit
3. by efficiency it must coincide with PFE demand

ETE where NA fails (slightly)

Category 2: observed market prices fail to satisfy NA: $q_L^0 \neq q^0 q^1$

- There is an $H-1$ dimensional set of this type (i.e., any efficient allocation can arise, like in the SFT)
- for any ETE allocation, prices can be set arbitrarily close to PFE prices.
- thus any small deviation from PFE might induce a major shift in the allocation of goods.

Key Idea for indeterminacy of Category 2

Key Observation

each household has forecasts which make it indifferent between holding two bonds: in particular, any budget feasible position on bonds is accepted as part of utility maximization

we want to construct an ETE -

- Fix ex post prices which are equal to PFE, except $q_L^0 \neq q^0 q^1$ but difference is arbitrarily small
- say $q_L^0 < q^0 q^1$: buying L by selling S will turn out to be profitable
- WTS: an ETE where household h consumes far more than in PFE
- NTS: there are forecasts and trading plans which make it budget feasible and utility maximizing

Rich by Accident - a variant of SFT

construction method

1. find optimistic forecasts which make the forecast income much higher than in PFE (e.g. high prices if his endowments are high in future)
2. find a profitable enough 'Sell S to buy L" strategy which gives such a higher income level in period 1
   - ♡ he is prepared to do this not because he thinks it is a profitable trade
3. also find appropriate forecasts for the other households which induce the intended income distribution

Notes: who gets richer?

- h is a loser in other ETE where h holds a pessimistic forecasts
- it is not the accuracy of forecasts, but optimism/pessimism which is not part of the primitives of the model
  - it appears as if optimism - but the agent does not know it is optimistic

Application - no trade result

- Suppose that the initial endowments are efficient
- it can be shown that there must be no trade in any of category 1 TE.
- epistemic interpretation - even if rational expectation is not assumed, there cannot be any speculative trade if no arbitrage condition is satisfied ex post.
- but the multiplicity result of category 2 still holds in this extreme economy
- epistemic interpretation: purely speculative trade is possible when there are "redundant assets"

A Behavioral Guide

- when there are multiple choices which are indifferent in your calculation, you should probably choose to minimize the loss from miscalculation
  - a possible link to "maximin approach" in decision theory
- do not take a large leveraged position in financial markets
- but of course if you want to be rich by accident then you might prefer to take a large position, which might make you poor by accident